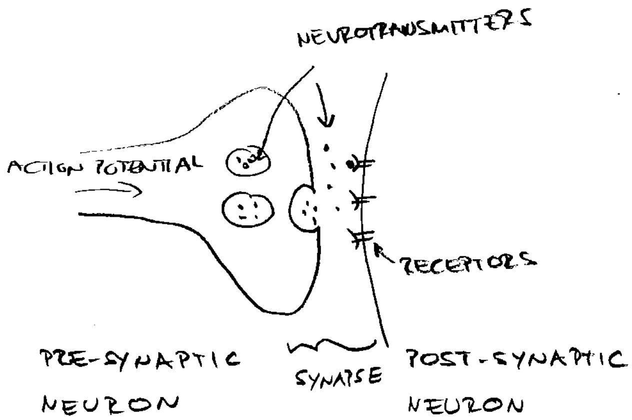
SYNAPSES: CONNECTING NEURONS INTO NETWORKS



(PRE-SYNAPTIC) ACTION POTENTIAL

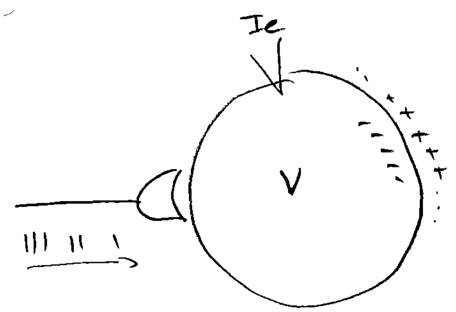
→ NEUROTRANSMITTER RELEASE

→ BINDS RECEPTORS

→ (POST-SYNAPTIC) ION CHANNELS OPEN

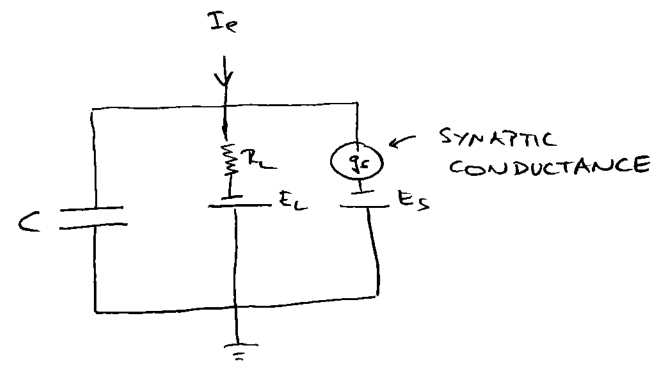
→ MEMBRANE POTENTIAL CHANGES

MODELING SYNAPSES



"ONE-COMPARTMENT MODEL NEURON"

$\hat{=}$



PASSIVE NEURON MODELS
(IGNORE ION CHANNELS)

$$Q = CV \Rightarrow \frac{dQ}{dt} = C\frac{dV}{dt}$$

$$V = RI \Rightarrow I = V/R = gV$$

$$I_e = I_c + I_R + I_s \quad \text{(KIRCHHOFF)}$$

$$I_e = C\frac{dV}{dt} + g_L(V - E_L) + g_s(V - E_s)$$

WHAT IS g_s?

$$g_s = \bar{g}_s P_s P_{REL}$$

$\bar{g}_s$: MAX CONDUCTANCE

P_s: FRACTION OF CHANNELS OPEN / PROB. CHANNEL OPENS W/ NEUROTRANSMITTER

P_{REL}: PROB. TRANSMITTER IS RELEASED GIVEN ACTION POTENTIAL

ASSUME $P_{REL} = 1$

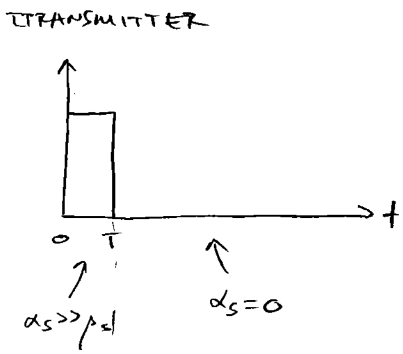
KINETIC MODEL OF POSTSYNAPTIC CHANNELS:

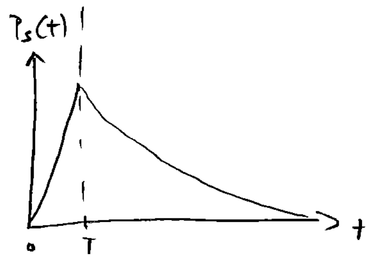
$$\text{CLOSED} \underset{\beta_s}{\overset{\alpha_s}{\rightleftharpoons}} \text{OPEN}$$

α_s: OPENING RATE ($\propto$ [TRANSMITTER]^n)

β_s: CLOSING RATE (CONSTANT)

$$\frac{dP_s}{dt} = \alpha_s(1-P_s) - \beta_s P_s$$

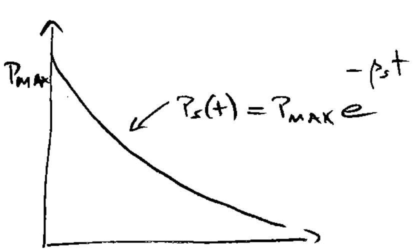




$0 \leq t \leq T$: $\quad \dfrac{dP_s}{dt} \approx \alpha_s(1-P_s)$, $P_s(0)=0 \Rightarrow P_s(t) = 1 - e^{-\alpha_s t}$

$t > T$: $\quad \dfrac{dP_s}{dt} = -\beta_s P_s \Rightarrow P_s(t) = P_s(T)\, e^{-\beta_s t}$

EXPONENTIAL IS A REASONABLE APPROXIMATION FOR SOME SYNAPSES:

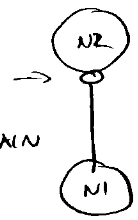


LINEAR FILTER MODEL OF A SYNAPSE

SYNAPSE b

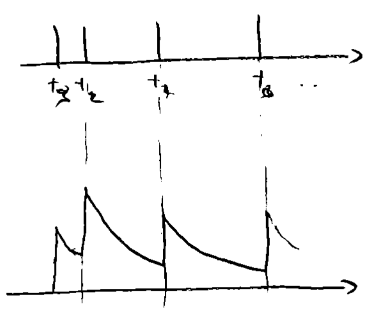
INPUT SPIKETRAIN

$s_b(t)$



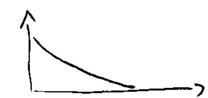
$$s_b(t) = \sum_i \delta(t-t_i)$$

DELTA FUNCTION: $\int_{-\infty}^{\infty} dt'\, \delta(t-t')\, f(t') = f(t)$



FILTER FOR SYNAPSE b: $K_b(t)$

OFTEN: $K_b(t) = P_{MAX}\, e^{-\beta_s t}$



$$g_b(t) = g_{b,MAX} \sum_{t_i < t} K(t-t_i)$$

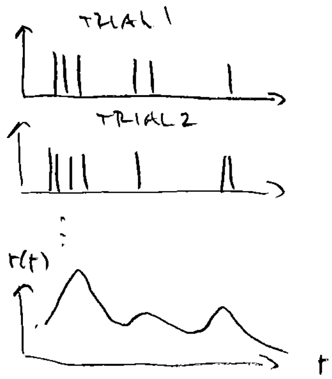
$$= g_{b,MAX} \int_{-\infty}^{t} dt'\, K(t-t')\, s_b(t')$$

FIRING RATE BASED MODELS

FIRING RATE:

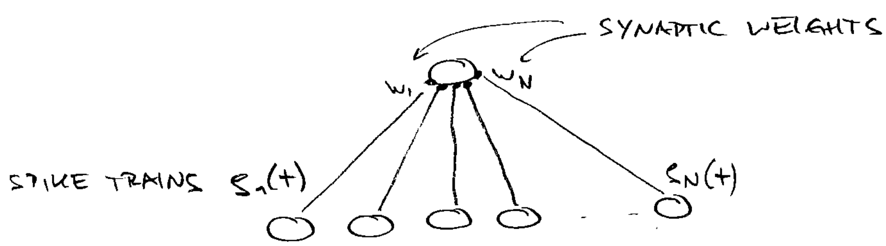
$$r(t) = \frac{1}{\Delta t} \int_{t}^{t+\Delta t} dt' \langle \rho(t') \rangle \qquad \rho(t): \text{ SPIKE TRAIN}$$

$r(t)\Delta t$: ~~AVG.~~ # OF SPIKES IN $(t, t+\Delta t)$ AVERAGED OVER TRIALS



$u(t)$: PRE-SYNAPTIC FIRING RATE /INPUT

$v(t)$: POST-SYNAPTIC FIRING RATE /OUTPUT



TOTAL SYNAPTIC CURRENT:

$$I_s(t) = \sum_{b=1}^{N} I_b(t)$$

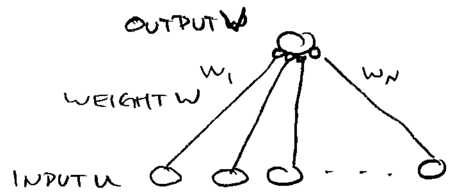
$$= \sum_{b=1}^{N} w_b \int_{-\infty}^{t} dt' \, K(t-t') \rho_b(t')$$

$$\approx \sum_{b=1}^{N} w_b \int_{-\infty}^{t} dt' \, K(t-t') \underline{\underline{u_b(t')}}$$

FIRING RATE!

SUPPOSE $k(t) = \frac{1}{\tau_s} e^{-t/\tau_s}$ $\quad \tau_s \sim 1/\beta$

$\Rightarrow$ SUBSTITUTE INTO $I_s(t)$, DIFFERENTIATE



$$\underline{\tau_s \frac{dI_s}{dt}} = -I_s + \sum_b w_b u_b = \underline{-I_s + \vec{w}\vec{u}}$$

MODEL FOR OUTPUT FIRING RATE

$$\underline{\tau_r \frac{dv}{dt} = -v + F(I_s(t))}$$

$\tau_r \ll \tau_s$: $\quad v = F(I_s(t)) \qquad F$: SIGMOID OR THRESHOLD

STATIC INPUT

$\underline{v_{ss} = F(I_s) = F(\vec{w}\cdot\vec{u})}$ $\quad$ (ARTIFICIAL NEURAL NETWORKS!)